Combined Effects of Suction/Injection on MHD Boundary Layer Flow of Nanofluid over a Horizontal Permeable Cylinder with Radiation

Shalini Jain, Dept. of Mathematics & Statistics, Manipal University Jaipur, Rajasthan, India.
E-mail: shalini.jain@jaipur.manipal.edu

Rakesh Choudhary, Dept. of Mathematics & Statistics, Manipal University Jaipur, Rajasthan, India.
E-mail: rakeshchoudhary@muj.manipal.edu

Abstract--- The present work is concerned with MHD boundary layer flow and heat transfer of Ag-water and $Al_{2}O_{3}$-water nanofluid over a horizontal stretching/shrinking cylinder. The investigation is carried out in the presence of suction/injection, heat source, radiation and shape of the nano-particles. The governing momentum and energy equations are transformed into nonlinear differential equations using suitable similarity transformations. These transformed equations are solved numerically using bvp4c Matlab solver. The effects of significant parameters such as nanofluid type, nano-particle volume friction, suction/injection parameter, magnetic field parameter, heat source parameter, radiation parameter on fluid flow and heat transfer are discussed.

Keywords--- MHD, Stretching/Shrinking Cylinder, Nanofluid, Suction/Injection, Heat Source, Radiation.

I. Introduction

studied by Zaimi et al. [19]. Effect of double stratification on mixed convection boundary-layer flow of a nanofluid past a vertical plate in a porous medium was examined by Srinivasacharya and Surender [20]. Recently, Mabood et al. [21] studied stagnation point flow of nanofluid over a moving plate with convective boundary condition and magnetohydro dynamics. Sulochana and Sandeep [22] analyzed stagnation-point flow and heat transfer behavior of Cu–water nanofluid towards horizontal and exponentially stretching/shrinking cylinders.


The present study is focused on a comparative study of shrinking/stretching conditions of the given setup in both suction and injection. This is the extended work of Sulochana and Sandeep [22]. Also, we studied the behavior of Ag-water nanofluid and $Al_2O_3$-water nanofluid embedded in horizontal stretching/shrinking cylinder.

The governing equations are transformed into ordinary differential equations using some similarity transformation. These equations are solved numerically with the help of bvp4c Matlab package. The effects of the various parameter such as curvature parameter $K$, magnetic field parameter $M$, radiation parameter $N$ etc. are examined and presents through suitable graphs and tables.

II. Mathematical Formulation

We have considered a steady stagnation point flow (fig. 1) and heat transfer of Ag-water and $Al_2O_3$-water nanofluid on R radius based horizontal stretching/shrinking cylinder engaged in an incompressible MHD viscous nanofluid with suction/injection and radiation at a constant temperature $T_w$.

![Fig 1: Physical Modal of the Problem](image)

In this study, we have done comparison study in different types of nanoparticles such that Ag-water and $Al_2O_3$-water. Table I shows the thermo-physical properties of the nanofluid. We have also considered that the free stream and suction/injection velocities are $u_x = ax / L$ and $u_w = cx / L$, where $L$ is a characteristic length and $a$, $c$ are both constants. Uniform heat source $Q$ is assumed in this study.

<table>
<thead>
<tr>
<th>Fluids</th>
<th>$\rho$ (kg m$^{-3}$)</th>
<th>$c_v$ (J kg$^{-1}$ K$^{-1}$)</th>
<th>$k$ (W m$^{-1}$ K$^{-1}$)</th>
<th>$\beta \times 10^6$ (K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_2O$ (Pure Water)</td>
<td>997.1</td>
<td>4179</td>
<td>0.613</td>
<td>21</td>
</tr>
<tr>
<td>$Ag$ (Silver)</td>
<td>10500</td>
<td>235</td>
<td>429</td>
<td>1.89</td>
</tr>
<tr>
<td>$Al_2O_3$ (Alumina)</td>
<td>3970</td>
<td>765</td>
<td>40</td>
<td>0.85</td>
</tr>
</tbody>
</table>
Under these assumption the governing boundary layer equations followed by [17] and [21], are:

\[
\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0, \quad (1)
\]

\[
\rho_{nf} \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} \right) = \mu_{nf} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) + u \frac{\partial v}{\partial x} + \sigma_{nf} B_0^2 (u_e - u), \quad (2)
\]

\[
(\rho_{nf})_{nf} \left( \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} \right) = k_{nf} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) - \frac{\partial q}{\partial r} + Q(T - T_\infty), \quad (3)
\]

Subject to boundary conditions are as follows:

\[
u = u_e, \quad v = v_e, \quad T = T_e, \quad \text{at} \quad r = R \quad \text{and} \quad u \to u_s, \quad T \to \infty \quad \text{as} \quad r \to \infty \quad (4)
\]

where \( u_e \) is the suction \((u_e < 0) \) and injection \((u_e > 0) \) velocity.

Where \( r \) is the coordinate measured in the radial direction, \( u \) and \( v \) are the velocity components in the \( x \) and \( r \) directions, respectively. \( \rho_{nf}, \mu_{nf} \) and \( \sigma_{nf} \) are the density, the dynamic viscosity and electrical conductivity of the fluid. \( B_0 \) is the imposed magnetic field strength. Further, \( T \) and \( T_\infty \) are the temperature in the boundary layer and free stream temperature, respectively, \( \rho \) is the isentropic heat capacitance of nanofluid, \( k \) is the effective thermal conductivity of the nanofluid, \( q \) is the radiative heat flux, \( Q \) is the heat source parameter, here we assumed \( Q = Q_0 \) for horizontal cylinder. The radiative heat flux in energy equation is given by Rossel and approximation and is denoted as follows \( q_r = -\frac{4\sigma T^4}{3k_1} \), where \( \sigma \) is the Stephan-Boltzmann constant and \( k_1 \) is the mean absorption constant. \( T^4 \) may be expressed in terms of \( T \) is \( T^4 = 4T_\infty T - 3T_\infty^4 \). The nanofluid constant are as follows [22]

\[
\frac{\rho_{nf}}{\rho} = (1 - \phi) \rho_f + \phi \rho_s, \quad \frac{\rho_{nf}}{\rho} = (1 - \phi) (\rho_{nf})_f + \phi (\rho_{nf})_s,
\]

\[
\frac{k_{nf}}{k} = \left( \frac{k_f + (n - 1) k_s - \phi(n - 1)(k_f - k_s)}{k_f + (n - 1) k_s + \phi(n - 1)(k_f - k_s)} \right), \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^3}, \quad (5)
\]

where \( n \) is the nanoparticle shape, \( n = 3/2 \) for cylindrical-shaped nanoparticle and \( n = 3 \) for spherical-shaped nanoparticles (Hamilton and Crosser 1962), \( \phi \) is the volume fraction of the nanoparticle. The subscripts \( f \) and \( s \) refer to fluid and solid properties, respectively. We introduce the following similarity transformation for Eqs. (1)-(3) with respect to the boundary conditions (Eq. (4))

\[
\eta = \frac{r^2 - R^2}{2R} \sqrt{\frac{a}{v_L}}, \quad \psi = \sqrt{\frac{v_L}{v_f}} \eta, \quad \theta(\eta) = \frac{T - T_\infty}{T_e - T_\infty}, \quad (6)
\]

where \( \eta \) is the similarity variable, \( \psi \) is the stream function defined as \( u = r^2 \frac{\partial \psi}{\partial r} \) and \( v = -r \frac{\partial \psi}{\partial x} \), these transformations are identically satisfied the Eq. (1). Using Eq. (6), Eqs. (2) and (3) reduces into the nonlinear ordinary differential equations (7) and (8)

\[
\frac{1}{(1 - \phi)^2} \left[ 1 + 2\eta K \right] f'' + 2Kf' + \left( 1 - \phi - \phi \frac{\rho_f}{\rho_s} \right) \left( ff'' - f'f' + M (1 - f') + 1 = 0 \quad (7) \right.
\]

\[
\frac{1}{Pr} \left( \frac{k_{nf}}{k_f} \right)^4 \left[ 1 + 2\eta K \right] f'' + 2Kf' + \left( 1 - \phi + \phi \frac{\rho_{nf}}{\rho_s} \right) \left( f f'' + f'f' + Q_H \theta \right) + M (1 - f') + 1 = 0 \quad (8)
\]

Subject to boundary conditions:

\[
f(0) = S, \quad f'(0) = \lambda, \quad \theta(0) = 1, \quad f'(\infty) \to 1, \quad \theta(\infty) \to 0, \quad (9)
\]

where \( K \) is the curvature parameter, \( Pr \) is the Prandtl number, \( N \) is the radiation parameter, \( Q_H \) is the heat source parameter, \( M \) is the magnetic field parameter, \( S \) is the suction \((S < 0) \) / injection \((S > 0) \) parameter and \( \lambda \) is the stretching/shrinking parameter, here \( \lambda > 0 \) for stretching and \( \lambda < 0 \) shrinking, such as
\[ K = \frac{1}{R} \sqrt{\frac{\nu_J L}{a}}, \quad Pr = \frac{\nu_J}{\alpha_f}, \quad Q_{II} = \frac{Q_L}{\alpha(\frac{\partial T}{\partial n})_f}, \quad M = \frac{\sigma_{\epsilon_f} B_L^2}{\alpha \rho_f}, \quad N = \frac{4\sigma T^3}{k^3}, \quad S = -v_c \sqrt{\frac{2v_J}{n}}, \quad \dot{\lambda} = \frac{c}{a}, \]  

(10)

The physical quantities of practical interest, in this study, local skin friction coefficient and local Nusselt number are defined as

\[ C_f = \frac{\tau_w}{\rho_f u_c^2}, \quad Nu = q_w - \frac{xq_w}{k_f(T_c - T_a)}. \]  

(11)

Where \( \tau_w \) and \( q_w \) are the shear stress and heat transfer of the surface of the cylinder. Using Eq. (6) we get

\[ \sqrt{Re} C_f = \frac{1}{(1 - \phi)\nu} f''(0), \quad \frac{Nu}{\sqrt{Re}} = -\frac{k_{sf}}{k_f} \left( 1 + \frac{4N}{3} \right) \theta'(0). \]  

(12)

Where \( Re \) is the local Reynolds number.

III. Result and Discussion

Using BVP4c Matlab package equations (7) and (8) are solved numerically with the given boundary conditions (Eq. (9)). We have considered \( Pr = 6.2, \quad \eta = 50, \quad K = 0.5, \quad M^2 = 0.5, \quad N = 1, \quad Q_{II} = 1, \quad \dot{\lambda} = 2, \quad n = 3 \) and \( S = 1 \) for numerical results. For whole computational study excluded the varied values as shown in particular figures and table, these values are preserved same.

Numerical results show the variation of the non-dimensional governing parameter such as curvature parameter \( K \), magnetic field parameter \( M \), radiation parameter \( N \), suction/injection parameter \( S \), heat source parameter \( Q_{II} \), shrinking/stretching parameter \( \dot{\lambda} \), nanoparticles volume fraction \( \phi \) and shape of the nanoparticles.

In this study, our main aim to study the comparison of shrinking/stretching parameter on magnetic field effect and radiation effect for considered setup. Till now there is no study have been dedicated for this setup.

Table 2 and table 3 depicts the comparison of the present results with the previous results of Bhattacharyya [14], Najib et al. [17] and Sulochana et al. [22]. This comparison shows that present results are very well agreement with the previous results. Also these results verify the validity of the present results.

Table 2: Comparison of the Values of \( f''(0) \) for Different Values of \( \dot{\lambda} \), when \( \phi = 0 \), \( K = 0 \) and \( M = 0 \)

<table>
<thead>
<tr>
<th>( \dot{\lambda} )</th>
<th>Bhattacharyya [14]</th>
<th>Najib et al. [17]</th>
<th>Present results</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.25</td>
<td>1.402240</td>
<td>1.402240</td>
<td>1.402247</td>
</tr>
<tr>
<td>-0.50</td>
<td>1.495669</td>
<td>1.495669</td>
<td>1.495630</td>
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<tr>
<td>-0.75</td>
<td>1.489298</td>
<td>1.489298</td>
<td>1.489280</td>
</tr>
<tr>
<td>-1.00</td>
<td>1.328816</td>
<td>1.328816</td>
<td>1.328791</td>
</tr>
</tbody>
</table>

Table 3: Comparison of the Values of \( f''(0) \) for Different Values of \( \dot{\lambda} \) and \( K \), when \( \phi = 0 \), \( M = 0 \)

<table>
<thead>
<tr>
<th>( \dot{\lambda} )</th>
<th>Najib et al. [17]</th>
<th>Sulochana et al. [22]</th>
<th>Present result</th>
<th>Najib et al. [17]</th>
<th>Sulochana et al. [22]</th>
<th>Present result</th>
</tr>
</thead>
<tbody>
<tr>
<td>When ( K = 0.2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.25</td>
<td>1.539615</td>
<td>1.539615</td>
<td>1.539560</td>
<td>1.667278</td>
<td>1.667278</td>
<td>1.667283</td>
</tr>
<tr>
<td>-0.50</td>
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<td>1.670569</td>
<td>1.670514</td>
<td>1.830752</td>
<td>1.830752</td>
<td>1.830751</td>
</tr>
<tr>
<td>-0.75</td>
<td>1.712534</td>
<td>1.712534</td>
<td>1.712451</td>
<td>1.911938</td>
<td>1.911938</td>
<td>1.911932</td>
</tr>
<tr>
<td>-1.00</td>
<td>1.629767</td>
<td>1.629767</td>
<td>1.629724</td>
<td>1.883619</td>
<td>1.883619</td>
<td>1.883601</td>
</tr>
</tbody>
</table>

Figures 2a, 2b, 2c and 2d shows the effects of curvature parameter \( K \) and comparison of shrinking/stretching parameter on both velocity profile and temperature profile for Ag-water nanofluid and \( Al_{2}O_{3} \)-water nanofluid. In figure 2a we compared shrinking/stretching parameter for various values of \( K \) when suction is applied.

It is clear that for increasing value of curvature parameter \( K \), velocity profile enhanced for both shrinking and stretching condition. Because when we increase the value of \( K \), the radius of the cylinder increases, this procedure helps in to decrease the contact area of the cylinder, hence velocity boundary layer thickness increases. On the other hand in figure 2b boundary layer thickness increases for stretching condition but for shrinking condition velocity...
profile decreases hence boundary layer thickness reduces. But in figures, 2c and 2d, due to above reason temperature profile increases with curvature parameter $K$ for both situations. Figure 3a, 3b, 3c and 3d displays the comparison of shrinking/stretching parameter $\lambda$ and effects of magnetic field parameter for Ag-water nanofluid and $Al_2O_3$-water nanofluid for both suction/injection on velocity profile and temperature profile. It is clearly seen that for every increasing value of magnetic field parameter $M$, velocity profile decreases for both the conditions. This is due to Lorentz force, creates a resistive type force, which is slow down the motion of fluid in the velocity boundary layer flow.

Hence velocity boundary layer thickness decreases. Due to the same reason enhanced the thermal boundary layer thickness, hence increase in magnetic field parameter, thermal boundary layer thickness increases. In figures 3a and 3b Ag-water nanofluid show high reduction compare to $Al_2O_3$-water nanofluid. But in temperature profile (see fig. 3c and 3d) Ag-water nanofluid shows high enhancement. Figure 4a, 4b, 4c and 4d illustrates the comparison of shrinking/stretching parameter $\lambda$ and effects of nanoparticle fluid friction $\phi$ in velocity and temperature profile for both Ag-water nanofluid and $Al_2O_3$-water nanofluid.

It is obvious from figures that improvement in volume friction of nanoparticles decreases the velocity profile for stretching condition and increases for shrinking, also enhances the thickness of thermal boundary layer flow. This is because of an improvement in fluid friction of nanoparticles shrinks the velocity boundary layer thickness appropriate to the friction near the walls.

This reduction is high for Ag-water nanofluid. Similarly, due to above reason helps to improve the thermal conductivity of the flow, hence increases in thermal boundary layer thickness for both cases suction/injection in shrinking and stretching (see figs. 4c and 4d). This hike in temperature profile is high for Ag-water nanofluid.

Figure 5a, 5b and 5c depicts the influence of suction/injection parameter $S$ in velocity and temperature profile for both fluids when shrinking and stretching respectively.

Negative values of $S$ shows suction, and positive values of $S$ indicates injection.

All the graphs shows reduction in velocity boundary layer thickness and thermal boundary layer thickness for increasing value of suction/injection parameter $S$ for both shrinking and stretching. Normally suction/injection parameter $S$ shrinks the velocity and thermal boundary layer thickness.

In velocity boundary layer profile Ag-water nanofluid show more reduction than $Al_2O_3$-water nanofluid, on the other hand in thermal boundary layer profile $Al_2O_3$-water nanofluid displays high reduction. Figure 6a, 6b and 6c illustrates the effects of stretching/shrinking parameter $\lambda$ in velocity and temperature profile.

It is observed from figures that an increase in stretching/shrinking parameter $\lambda$ enhance the velocity boundary layer thickness and reduces the fluid temperature for both suction and injection. Here negative values of $\lambda$ indicate shrinking of the cylinder while positive value shows stretching of cylinder. Figures 7a and 7b displays the comparison of shrinking/stretching parameter $\lambda$ and effects of radiation parameter $N$ on both fluids for both injection and suction respectively. We know that higher values of radiation parameter produce heat in the fluid, therefore an increase in radiation parameter $N$, increases thermal boundary layer thickness for both suction and injection.

Figures 8a and 8b illustrate the comparison in shrinking and stretching and effects of heat source parameter $Q_H$ in temperature profile for both fluids for injection and suction respectively. Generally, we know that heat source parameter enhanced the thermal boundary thickness, hence for increasing values of heat source parameter, increases thermal boundary layer thickness for both conditions. Also it is seen that Ag-water nanofluid shows enhancement than $Al_2O_3$-water nanofluid.

Figures 9a and 9b depicts the comparison of shrinking and stretching for nanoparticle shapes of the temperature profile for Ag-water nanofluid and $Al_2O_3$-water nanofluid for suction and injection.

It is clear from the figure that for spherical-shaped nanoparticle’s heat transfer enhancement is more than the cylindrical-shaped nanoparticles for both fluids on suction and injection.
Table 4: Variation in Skin Friction Coefficient for Different Values of \( K, M, S \) and \( \lambda \) when \( \phi = 0.1 \)

<table>
<thead>
<tr>
<th>( K )</th>
<th>( M )</th>
<th>( S )</th>
<th>( \lambda )</th>
<th>( \frac{1}{(1-\phi)^2}f''(0) ) ( \text{Ag-Water} )</th>
<th>( \frac{1}{(1-\phi)^2}f''(0) ) ( \text{Al}_2\text{O}_3\text{-Water} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>-3.431488</td>
<td>-2.517434</td>
</tr>
<tr>
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</tr>
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<td>-2.517434</td>
</tr>
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<td>1</td>
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<td>2</td>
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<td>0.5</td>
<td>2</td>
<td>0.626112</td>
<td>0.778909</td>
</tr>
</tbody>
</table>

Table 5: Variation Rate of Heat Transfer for Different Values of \( K, M, S, \lambda, N, Q_H \) and \( n \) when \( \phi = 0.1 \) and \( \text{Pr} = 6.2 \)

<table>
<thead>
<tr>
<th>( K )</th>
<th>( M )</th>
<th>( S )</th>
<th>( \lambda )</th>
<th>( N )</th>
<th>( Q_H )</th>
<th>( n )</th>
<th>( \frac{k_{sf}}{k_f} \left( 1+\frac{4N}{3} \right) \theta'(0) ) ( \text{Ag-Water} )</th>
<th>( \frac{k_{sf}}{k_f} \left( 1+\frac{4N}{3} \right) \theta'(0) ) ( \text{Al}_2\text{O}_3\text{-Water} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>4.573741</td>
<td>4.984970</td>
</tr>
<tr>
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<td>3</td>
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<td>4.984970</td>
</tr>
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<td>1</td>
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<td>2.832423</td>
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Table 4 and 5 shows the variation of skin friction coefficient and rate of heat transfer for different parameters such as curvature parameter $K$, magnetic field parameter $M$, suction/injection parameter $S$, stretching/shrinking parameter $\lambda$, radiation parameter $N$, heat source parameter $Q_H$ and nanoparticles shaped $n$ when $\phi = 0.1$ and $Pr = 6.2$. 

Figures 2a, 2b, 3a, 3b, 3c, 3d, 4a, 4b: Comparisons of shrinking/stretching parameter when $S=1$ on temperature profile for $K$, $M$, $n$, $\phi$. 

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IV. Conclusion

The present study, we have investigated the combined effects of suction/injection on MHD boundary layer flow of nanofluid over a horizontal permeable cylinder with radiation. We have also compared the shrinking and stretching condition for this setup. Influence of various parameters such as curvature parameter $K$, magnetic field parameter $M$, suction/injection parameter $S$, shrinking/stretching parameter $\lambda$, radiation parameter $N$ and heat source parameter $Q_H$ are examined and displayed through suitable graphs and tables. Some important results are as follows: (a) An increment in magnetic field parameter $M$, causes velocity boundary layer thickness decreases, whereas thermal boundary layer thickness increases for both suction/injection and shrinking/stretching conditions.
(b) When we increase radiation parameter $N$ and heat source parameter $Q_u$, thermal boundary layer thickness shows enhancement for both conditions.

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**References**


**Dr. Shalini Jain**, Professor, Department of Mathematics and Statistics, is presently working in Manipal University. I was born and brought in district Jaipur, Rajasthan, India. In the journey of teaching and learning, I completed my M.Sc., M Phil. and Ph. D. in Mathematics from the University of Rajasthan. Since then, I have a progressive career of sixteen years in teaching math. I have performed well as a faculty member and have a keen interest in research. Research paper publication and presentation is a quest for more and more and deeper knowledge of my subject area. I have published several research papers in good journals of reputed including which includes Procedia Engineering (Elsevier), Int. Journal of Energy and Technology (USA). Supervising Research Scholars.

**Rakesh Choudhary**, was born and brought up in district Jaipur, Rajasthan, India. I have done my B.Sc. (PCM) from University Maharaja College, Jaipur in 2012 and M.Sc. (Mathematics) from Manipal University Jaipur in 2014. Presently I am pursuing my Ph.D. from Manipal University Jaipur in Fluid Dynamics and working as teaching assistant/Research Scholar in Manipal University Jaipur. I have attended a number of national and international conferences & presented papers. I have published research papers in journals including Global and Stochastic Analysis and Procedia Engineering (Elsevier).